Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number

Candidate Number

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Friday 20 May 2016 – Morning
Time: 1 hour 30 minutes

Paper Reference
6667/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1. Given that k is a real number and that

$$\mathbf{A} = \begin{pmatrix} 1+k & k \\ k & 1-k \end{pmatrix},$$

find the exact values of k for which $\mathbf{A}$ is a singular matrix. Give your answers in their simplest form.

(Total 3 marks)

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2. $\mathrm{f}(x) = 3x^{\frac{3}{2}} - 25x^{-\frac{1}{2}} - 125 \quad x > 0.$

(*a*) Find $\mathrm{f}'(x)$

(2)

The equation $\mathrm{f}(x) = 0$ has a root α in the interval [12, 13].

(*b*) Using $x_0 = 12.5$ as a first approximation to α, apply the Newton-Raphson procedure once to $\mathrm{f}(x)$ to find a second approximation to α, giving your answer to 3 decimal places.

(4)

(Total 6 marks)

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3. (*a*) Using the formula for $\sum_{r=1}^{n} r^2$ write down, in terms of n only, an expression for

$$\sum_{r=1}^{3n} r^2 .$$

(1)

(*b*) Show that, for all integers n, where $n > 0$,

$$\sum_{r=2n+1}^{3n} r^2 = \frac{n}{6}(an^2 + bn + c)$$

where the values of the constants a, b and c are to be found.

(4)

(Total 5 marks)

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4. $$z = \frac{4}{1+\mathrm{i}}.$$

Find, in the form $a + \mathrm{i}b$ where $a, b \in \mathbb{R}$,

(*a*) z,

(2)

(*b*) z^2.

(2)

Given that z is a complex root of the quadratic equation $x^2 + px + q = 0$, where p and q are real integers,

(*c*) find the value of p and the value of q.

(3)

(Total 7 marks)

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5. Points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$, where $p^2 \neq q^2$, lie on the parabola $y^2 = 4ax$.

(*a*) Show that the chord PQ has equation

$$y(p + q) = 2x + 2apq.$$

(5)

Given that this chord passes through the focus of the parabola,

(*b*) show that $pq = -1$.

(1)

(*c*) Using calculus find the gradient of the tangent to the parabola at P.

(2)

(*d*) Show that the tangent to the parabola at P and the tangent to the parabola at Q are perpendicular.

(2)

(Total 10 marks)

6.

$$\mathbf{P} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

(*a*) Describe fully the single geometrical transformation U represented by the matrix **P**.

(2)

The transformation U maps the point A, with coordinates (p, q), onto the point B, with coordinates $(6\sqrt{2}, 3\sqrt{2})$.

(*b*) Find the value of p and the value of q.

(3)

The transformation V, represented by the 2×2 matrix **Q**, is a reflection in the line with equation $y = x$.

(*c*) Write down the matrix **Q**.

(1)

The transformation U followed by the transformation V is the transformation T. The transformation T is represented by the matrix **R**.

(*d*) Find the matrix **R**.

(3)

(*e*) Deduce that the transformation T is self-inverse.

(1)

(Total 10 marks)

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7. A complex number z is given by

$$z = a + 2\text{i},$$

where a is a non-zero real number.

(a) Find $z^2 + 2z$ in the form $x + \text{i}y$ where x and y are real expressions in terms of a.

(4)

Given that $z^2 + 2z$ is real,

(b) find the value of a.

(1)

Using this value for a,

(c) find the values of the modulus and argument of z, giving the argument in radians, and giving your answers to 3 significant figures.

(3)

(d) Show the points P, Q and R, representing the complex numbers z, z^2 and $z^2 + 2z$ respectively, on a single Argand diagram with origin O.

(3)

(e) Describe fully the geometrical relationship between the line segments OP and QR.

(2)

(Total 13 marks)

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8. (i) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2} = 1 - \frac{1}{(n+1)^2}.$$

(5)

(ii) A sequence of positive rational numbers is defined by

$$u_1 = 3,$$
$$u_{n+1} = \frac{1}{3}u_n + \frac{8}{9}, \quad n \in \mathbb{Z}^+.$$

Prove by induction that, $n \in \mathbb{Z}^+$,

$$u_n = 5 \times \left(\frac{1}{3}\right)^n + \frac{4}{3}.$$

(5)

(Total 10 marks)

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9. The rectangular hyperbola, H, has cartesian equation $xy = 25$.

(a) Show that an equation of the normal to H at the point $P\left(5p, \frac{5}{p}\right)$, $p \neq 0$,

$$y - p^2x = \frac{5}{p} - 5p^3.$$

(5)

This normal meets the line with equation $y = -x$ at the point A.

(b) Show that the coordinates of A are

$$\left(-\frac{5}{p} + 5p, \frac{5}{p} - 5p\right).$$

(3)

The point M is the midpoint of the line segment AP.
Given that M lies on the positive x-axis,

(c) find the exact value of the x coordinate of point M.

(3)

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

Question Number	Scheme	Marks
1.	Determinant of **A** $= (1-k)(1+k) - k^2 = 0$	M1
	$1-k+k-k^2-k^2 (=0)$ $1-2k^2(=0)$ So $k = \dfrac{\pm\sqrt{2}}{2}$	A1 A1 (3)
		3 marks

Notes

M1: for attempting $ad - bc = 0$ with $'=0'$ seen or **used** later in the solution.

A1: Correct (unsimplified) expression on LHS or correct equation after brackets expanded.

A1: Accept $\pm\dfrac{\sqrt{2}}{2}$, $\pm\dfrac{1}{\sqrt{2}}$, $\pm\sqrt{\dfrac{1}{2}}$, $\pm\sqrt{0.5}$. Must have $\pm$ for mark.

Question Number	Scheme	Marks
2.	(a) $f'(x) = \frac{9}{2}x^{\frac{1}{2}} + \frac{25}{2}x^{-\frac{3}{2}}$	M1 A1 (2)
	(b) f(12.5)= 0.5115… (at least 0.51…) and $f'(12.5) = 16.1927...$ (at least 16… seen) $x_1 = 12.5 - \frac{f(12.5)}{f'(12.5)} = 12.5 - \frac{0.5115}{16.1927...} = 12.468$	B1, B1 M1 A1 (4)
		6 marks

Notes

(a) M1: for attempting differentiation i.e. decrease a power by 1
A1 Accept equivalent expression i.e. condone equivalent fractions.

(b) B1: One correct, must be explicitly seen if final answer incorrect, may be implied by correct final answer.
B1: Both correct; must be explicitly seen if final answer incorrect, may be implied by correct final answer.
M1:for attempting Newton- Raphson with their values for f(12.5) and $f'(12.5)$
A1: cao correct to 3dp
Newton Raphson used more than once – isw.

Question Number	Scheme	Marks
3.	(a) $\sum_{r=1}^{3n} r^2 = \frac{1}{6}3n(3n+1)(6n+1)$ or $\sum_{r=1}^{3n} r^2 = \frac{1}{2}n(3n+1)(6n+1)$ or equivalent	B1 (1)
	(b) See $\sum_{r=1}^{2n} r^2 = \frac{1}{3}n(2n+1)(4n+1)$ or equivalent	B1
	Attempt to use $\sum_{r=1}^{3n} r^2 - \sum_{r=1}^{2n} r^2 = \frac{n}{6}\{3(3n+1)(6n+1)-2(2n+1)(4n+1)\}$	M1
	$=\frac{n}{6}\{(54n^2+27n+3)-(16n^2+12n+2)\}$	dM1
	$=\frac{n}{6}\{(38n^2+15n+1)\}$	A1
	$(a=38, b=15, c=1)$	
		(4)
		5 marks

Notes

(a) B1: Either right hand side or exact equivalent - isw if expanded

(b) B1: States or uses $\sum_{r=1}^{2n} r^2 = \frac{1}{3}n(2n+1)(4n+1)$

M1: Subtracts their sum to $2n$ or $2n-1$ **and** attempts to factorise by $\frac{n}{6}$ seen anywhere.

dM1: Expands two quadratics dependent on first M1

A1: cao

Question Number	Scheme	Marks
4.	(a) $z = \frac{4(1-\mathrm{i})}{(1+\mathrm{i})(1-\mathrm{i})}$	M1
	$z = 2(1-\mathrm{i})$ or 2 – 2i or exact equivalent.	A1 (2)
	(b) $z^2 = (2-2\mathrm{i})(2-2\mathrm{i}) = 4-8\mathrm{i}+4\mathrm{i}^2$	M1
	$= -8\mathrm{i}$	A1 cao (2)
	(c) If z is a root so is z^* So $(x - 2 + 2\mathrm{i})(x - 2 - 2\mathrm{i})$ (or $x^2 - 2\mathrm{Re}(z).x + \|z\|^2$)	M1
	So $(x - 2 + 2\mathrm{i})(x - 2 - 2\mathrm{i}) = 0$ (or $x^2 - 2\mathrm{Re}(z).x + \|z\|^2 = 0$) and so $p = q =$	M1
	Equation is $x^2 - 4x + 8(=0)$ or $p = -4$ and $q = 8$	A1 (3) (7 marks)
ALT 1	(c) Substitutes $z = 2 - 2\mathrm{i}$ and $z^2 = -8\mathrm{i}$ into quadratic and equates real and imaginary parts to obtain $2p + q = 0$ and $-2p - 8 = 0$	M1
	Attempts to solve simultaneous equations to obtain $p = -4$ and $q = 8$	M1A1
ALT 2	(c) Attempts to obtain $p = -$ sum of roots	M1
	Attempts product of roots to obtain $q=$	M1
	Equation is $x^2 - 4x + 8(=0)$ or $p = -4$ and $q = 8$	A1
ALT 3	(c) $x - 2 = \pm 2\mathrm{i}$ either sign acceptable	M1
	$(x-2)^2 = -4 \Rightarrow x^2 - 4x + 4 = -4$ i.e square and attempt to expand to give 3-term quadratic	M1
	Equation is $x^2 - 4x + 8(=0)$ or $p = -4$ and $q = 8$	A1

Notes

(a) M1: Multiplies numerator and denominator by 1 – i or by -1 + i
A1: cao
(b) M1: Squares their z, or the given $z = \frac{4}{1+\mathrm{i}}$, to produce at least 3 terms which can be implied by the correct answer.
A1: $-8\mathrm{i}$ or $0-8\mathrm{i}$ only
(c) M1: Uses their z and z^* in $(x-z)(x-z^*)$
M1: Multiplies two factors and obtains $p =$ or $q =$
A1: Both correct required – can be implied by $x^2 - 4x + 8$
ALT 1
(c) M1: Substitutes their z and their z^2 into the quadratic and equates real and imaginary parts to obtain two equations in p and q
M1: Attempts to solve for one unknown to obtain $p =$ or $q =$
A1: Both correct required – can be implied by $x^2 - 4x + 8(=0)$

Question Number	Scheme	Marks
5.	(a) Gradient $=\dfrac{2ap-2aq}{ap^2-aq^2}$ seen	B1
	$\left(\dfrac{2ap-2aq}{ap^2-aq^2}=\dfrac{2a(p-q)}{a(p-q)(p+q)}\right)=\dfrac{2}{p+q}$ or seen in an equation	B1
	uses $y-y_1=m(x-x_1)$ to give $(y-2aq)="m"(x-aq^2)$ or equivalent with p or uses $y = mx + c$ to give $y="m"x+c$ and substitute a point to find c or uses $\dfrac{y-y_1}{y_2-y_1}=\dfrac{x-x_1}{x_2-x_1}$ to give $\dfrac{(y-2aq)}{2ap-2aq}=\dfrac{(x-aq^2)}{ap^2-aq^2}$ or equivalent with p	M1
	so $(y-2aq)=\dfrac{2}{p+q}(x-aq^2)$ or $(y-2ap)=\dfrac{2}{p+q}(x-ap^2)$ or $y=\dfrac{2}{p+q}x+\dfrac{2apq}{p+q}$ or $\dfrac{(y-2aq)}{2a}=\dfrac{(x-aq^2)}{a(p+q)}$	A1
	See $2aq^2$ or $2ap^2$ term appear and disappear to give $y(p+q)=2x+2apq$ *	A1 cso (5)
	(b) Substitute $(a , 0)$ into line equation, to give $0=2a+2apq$ so $pq=-1$	B1 (1)
	(c) $y=2a^{\frac{1}{2}}x^{\frac{1}{2}}\Rightarrow\dfrac{dy}{dx}=a^{\frac{1}{2}}x^{-\frac{1}{2}}$ or $y^2=4ax\Rightarrow 2y\dfrac{dy}{dx}=4a$ or $\dfrac{dy}{dx}=\dfrac{dy}{dp}\times\dfrac{dp}{dx}\Rightarrow\dfrac{dy}{dx}=2a\times\dfrac{1}{2ap}$	M1
	So at P tangent gradient $=\dfrac{1}{p}$	A1 (2)
	(d) At Q tangent gradient $=\dfrac{1}{q}$	B1
	$\dfrac{1}{p}\times\dfrac{1}{q}=\dfrac{1}{pq}=\dfrac{1}{-1}=-1$ with at least one intermediate step, the tangents are **perpendicular** or at **right angles**	B1cso (2) (10 marks)

Notes

(a) B1: Correct statement for gradient (isw) B1: $\dfrac{2}{p+q}$ - can be seen later in the solution.

M1: use of a correct formula for a line equation through P or Q with their gradient.
Must be finding a chord, not a tangent or a normal.

A1: for a correct line equation with simplified gradient in any equivalent form

A1: cso (as given answer)

(b) B1: For using $(a, 0)$ to show that $pq = -1$

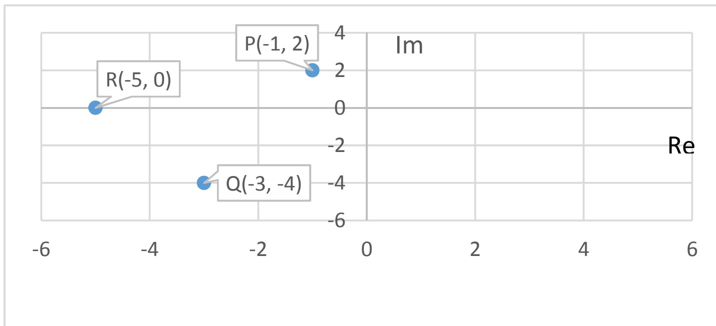
(c) M1: Use calculus to find an expression for dy/dx and substitute coordinates of P.
They may use chord gradient and let p tend to q.

(d) B1: $1/q$ seen B1: $\dfrac{1}{p}\times\dfrac{1}{q}=-1$ or $\dfrac{1}{p}=-\dfrac{1}{1/q}$ or $\dfrac{1}{q}=-\dfrac{1}{1/p}$ and at least words in bold with no errors seen.

Question Number	Scheme	Marks
6.	(a) **Rotation** ,135 degrees or $\frac{3\pi}{4}$ radians (anticlockwise) about O or 225 degrees or $\frac{5\pi}{4}$ clockwise about O.	M1, A1 (2)
	(b) $\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 6\sqrt{2} \\ 3\sqrt{2} \end{pmatrix}$	
	$-p-q=12$ and $p-q=6$ or equivalent	M1A1
	$p=-3$ and $q=-9$ or $\begin{pmatrix} -3 \\ -9 \end{pmatrix}$	B1 cso (3)
	ALT Uses Inverse matrix $\mathbf{P}^{-1}$ with vector $= \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 6\sqrt{2} \\ 3\sqrt{2} \end{pmatrix}$	M1A1
	$p=-3$ and $q=-9$ or $\begin{pmatrix} -3 \\ -9 \end{pmatrix}$	B1 cso (3)
	(c) $\mathbf{Q} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	B1 (1)
	Accept **T** if used instead of **R**	
	(d) $\mathbf{R} = "\mathbf{Q}"\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$	M1 A1 A1 (3)
	(e) $\mathbf{R}^{-1} = \frac{1}{-1}\begin{pmatrix} -\frac{1}{\sqrt{2}} & +\frac{1}{\sqrt{2}} \\ +\frac{1}{\sqrt{2}} & +\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$	
	$= \mathbf{R}$ (so matrix is self inverse and so transformation is self inverse)	B1 **(1)** **10 marks**
	ALT 1 (e) $\mathbf{RR} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (so **R** is self inverse and so transformation is self inverse)	B1
	ALT 2 (e) Matrix represents a reflection (so is self inverse)	B1

Notes

(a) M1: Rotation only A1: 135 degrees about O
SC: 135 degrees about O only award M1A0.
(b) M1: Multiplies matrices in correct order to obtain two equations in p and q.
A1: Two correct equations
B1 cso: p and q both correct, may be in vector form. No errors seen in solution.
ALT (b) M1: Attempt to find Inverse Matrix and pre-multiply A1: Correct Inverse Matrix used
B1 cso: p and q both correct, may be in vector form. No errors seen in solution.
(d) M1: Sets matrix product correct way round and obtains one correct term for their **Q**
A1: Two correct terms from a correct **Q.** **Q** incorrect award A0 here. A1: Completely correct matrix
(e) B1: Calculates $\mathbf{R}^{-1}$ and indicates that $\mathbf{R}^{-1} = \mathbf{R}$ or calculates $\mathbf{R}^2$ and indicates that $\mathbf{R}^2 = \mathbf{I}$ or states that **R** represents a reflection.

Question Number	Scheme	Marks
7.	(a) $z^2 = (a+2\text{i})(a+2\text{i}) = (a^2-4)+4\text{i}a$ So $z^2+2z = (a^2-4+2a)+\text{i}(4a+4)$ or $x=(a^2+2a-4)$ and $y=4a+4$	M1 M1 A1 A1 (4)
	(b) and so $4a+4=0 \to a=-1$	B1 (1)
	ALT (b) Substitute $a=-1$ and show that $y=0$	B1
	(c) $\lvert z \rvert = \sqrt{5}$ or awrt 2.24 $\arctan, (-2) = 2.03$	B1 M1, A1 cao (3)
	(d) 	M1 A1 B1ft (3)
	(e) OP and QR are parallel , and QR is twice the length of OP Or Enlargement with Scale Factor 2 (centre O), followed by translation $\begin{pmatrix}-3\\-4\end{pmatrix}$ Or Enlargement with Scale Factor 2, centre (3,4) or centre 3 + 4i $\overrightarrow{QR} = 2\overrightarrow{OP}$ with clear indication of vectors award B1B1, without vectors award B0B1	B1, B1 (2) **13 marks**

Notes:

(a) M1: Squares z to produce at least 3 terms which can be implied by the correct answer.
M1: Adds $2z$ to their z^2
A1: Correct x A1 Correct y accept $4a\text{i}+4\text{i}$
(b) B1: Completely accurate cao
(c) B1: √5 or 2.24 or awrt 2.24
M1 for using tan or arctan
A1 cao 2.03
(d) M1: **Either** their OP in the correct quadrant labelled P or z or their $-1+2\text{i}$ or their (-1,2) or axes labelled **or** their OQ in the correct quadrant labelled Q or z^2 or their $-3-4\text{i}$ or their (-3, -4) or axes labelled
A1: Both OP and OQ correct i.e. in the 2nd and 3rd quadrants respectively.
B1ft: $OR : z^2+2z(=-5)$ **on real axis** to left of the origin.
Accept points or lines. Arrows not required. Axes need not be labelled Re and Im.
Treat correct quadrant (or on axis) as important aspect for accuracy, lengths of lines if present can be accepted as correct.

Question Number	Scheme	Marks
8.	(i) If $n=1$, $\sum_{r=1}^{n}\frac{2r+1}{r^2(r+1)^2}=\frac{3}{4}$ **and** $1-\frac{1}{(n+1)^2}=\frac{3}{4}$**,** so true for n = 1.	B1
	Assume result true for $n = k$ **and** consider $\sum_{r=1}^{k+1}\frac{2r+1}{r^2(r+1)^2}=1-\frac{1}{(k+1)^2}+\frac{2(k+1)+1}{(k+1)^2(k+2)^2}$	M1
	$=1-\left(\frac{(k+2)^2}{(k+1)^2(k+2)^2}-\frac{2(k+1)+1}{(k+1)^2(k+2)^2}\right)=1-\left(\frac{(k^2+2k+1)}{(k+1)^2(k+2)^2}\right)$	A1
	$=1-\left(\frac{(k+1)^2}{(k+1)^2(k+2)^2}\right)=1-\left(\frac{1}{(k+1+1)^2}\right)$	M1
	True for ***n* = *k* + 1** if **true** for ***n* = *k***, (and **true** for ***n* = 1**) so **true** by induction for all $\boldsymbol{n\in Z^+}$	A1cso (5)
	(ii) n = 1: $u_1=5\times\left(\frac{1}{3}\right)^1+\frac{4}{3}=3$ so expression for u_n true for n = 1	B1
	Assume result true for $n = k$ **and** consider $u_{k+1}=\frac{1}{3}(5\times\left(\frac{1}{3}\right)^k+\frac{4}{3})+\frac{8}{9}$	M1
	Obtain $u_{k+1}=5\times\left(\frac{1}{3}\right)^{k+1}+\frac{4}{9}+\frac{8}{9}$	A1
	$5\times\left(\frac{1}{3}\right)^{k+1}+\frac{4}{3}$ and deduce that result is true for $n = k + 1$	dM1
	True for ***n* = *k* + 1** if **true** for ***n* = *k***, (and **true** for ***n* = 1**) so **true** by induction for all $\boldsymbol{n\in Z^+}$	A1 cso (5) **10 marks**

Notes:

(i) B1: Checks n = 1 on both sides **and** states true for n = 1 seen anywhere

M1: (Assumes true for) $n = k$ **and** adds $(k+1)^{\text{th}}$ term to sum of k terms

A1: $1-\left(\frac{(k^2+2k+1)}{(k+1)^2(k+2)^2}\right)$ seen (linked to 2nd M)

M1: $(k+1)^2(k+2)^2$ attempted as common denominator of two fractions.

A1cso: Makes correct complete induction statement including at least statements in bold. Accept $n\ge 1$ or $n=1,2,3...$ or all positive Integers or all n. Statement true for n = 1 here could contribute to B1 mark earlier.

(ii) B1: Checks n = 1 in u_n and states true for n = 1 seen anywhere.

M1: (Assumes result for) $n = k$ **and** substitutes u_k into correct expression for u_{k+1}

A1: $\frac{4}{9}+\frac{8}{9}$ or $\frac{1}{3}.\frac{4}{3}+\frac{8}{9}$ seen

dM1: Obtains $5\times\left(\frac{1}{3}\right)^{k+1}+\frac{4}{3}$ and statement true for $n = k + 1$ or equivalent seen anywhere dependent on previous M.

A1cso: Makes correct complete induction statement including at least statements in bold. Accept $n\ge 1$ or $n=1,2,3...$ or all positive Integers or all n. Statement true for n = 1 here could contribute to B1 mark earlier.

Question Number	Scheme	Marks
9 (a)	$y=\frac{25}{x}\Rightarrow\frac{dy}{dx}=-25x^{-2}$,	B1
	or $y+x\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=-\frac{y}{x}$ **or** $\dot{x}=5,\ \dot{y}=-\frac{5}{t^2}$ so $\frac{dy}{dx}=-\frac{1}{t^2}$	
	and at P $\frac{dy}{dx}=-\frac{1}{p^2}$ so gradient of normal is p^2	M1 A1
	Either $y-\frac{5}{p}=p^2(x-5p)$ **or** $y=p^2x+k$ and use $x=5p,\ y=\frac{5}{p}$	M1
	$\Rightarrow\quad y-p^2x=\frac{5}{p}-5p^3$ (*)	A1cso (5)
(b)	At the point A: $y+p^2y=\frac{5}{p}-5p^3$ or $-x-p^2x=\frac{5}{p}-5p^3$	
	$y(1+p^2)=\frac{5}{p}(1-p^4)$ or $-x(1+p^2)=\frac{5}{p}(1-p^4)$	M1
	$y=\frac{\frac{5}{p}(1-p^2)(1+p^2)}{(1+p^2)}=\frac{5}{p}(1-p^2)=\frac{5}{p}-5p$ or $x=\frac{-\frac{5}{p}(1-p^2)(1+p^2)}{(1+p^2)}=\frac{-5}{p}(1-p^2)=\frac{-5}{p}+5p$ *	M1
	so $x=-\frac{5}{p}(1-p^2)=-\frac{5}{p}+5p$ and $y=\frac{5}{p}-5p$ *	A1cso (3)
(c)	M has coordinates $\left(-\frac{5}{2p}+5p,\ \frac{5}{p}-\frac{5p}{2}\right)$ o.e.	B1
	So when $y=0$, $\frac{5}{p}-\frac{5p}{2}=0$ and $p=\sqrt{2}$ so M has x coordinate $\frac{15}{4}\sqrt{2}$ o.e.	M1 A1 (3)
		11 marks

Notes

(a) B1: Any correct expression for gradient of tangent
M1: Substitutes values into **derived expression using calculus** to give gradient of normal at P
A1: cao. Can be implied by use in equation of a straight line
M1: Use of formula for the equation of a straight line with their changed gradient
A1: cso
(b) M1: Replaces x by $-y$ or y by $-x$
M1: Factorises $(1-p^4)$ to simplify answer in first variable
A1 cso: Obtains both x and y
ALT (b) Accept Verification.
M1: Substitutes the coordinates of A into the equation of the normal
M1: Substitutes the coordinates of A into both the normal and $y = -x$.
A1 cso: No errors seen
(c) B1: Correct x- coordinate of midpoint (may be implied) and correct y coordinate, accept equivalent forms
M1: Puts their $y = 0$ and finds value for p to use in $x =$
A1: $+\frac{15}{4}\sqrt{2}$ or equivalent only